Chapter 478

Minimum Spanning Tree

Introduction

A minimum spanning tree links all nodes (points or vertices) of a network with the minimum length of all arcs. This procedure finds the minimum spanning tree of a network using a greedy algorithm. If the network is not connected, the solution, called a *minimum spanning forest*, is a combination of minimum spanning trees formed on the connected subsets.

The algorithm is used in applications such as transportation networks, computer networks, and water networks where a tree connecting all nodes must have minimum length.

Greedy Algorithm

The algorithm proceeds as follows:

1. Start with any node.

2. Connect this node to its nearest neighbor using any of the available branches.

3. Find the unconnected node that is closest any of the connected nodes. Connect these nodes.

4. Repeat steps 2 and 3 until all nodes have been connected.

Data Structure

This procedure requires a special data format in which the rows represent the network branches. Each branch is defined by two nodes and the distance between its nodes.

Consider the following example. Each row represents an electric cable that connects two orchards on a large fruit farm that produces many types of fruit. The nodes are identified by the type of fruit grown in that section. The length of the cable is given in the last column. These data are stored in the dataset *Wiring Network*.

Wiring Network Dataset

Site1	Site2	Length
A1	A2	6.8
A1	A3	3.2
A2	B1	9.1
A2	A3	8.5
A2	B2	7.2
B1	B2	10.3
A3	B2	14.8
A3	C1	5.6
B2	C1	8.2
B2	C2	9.1
C1	C2	10.9
Z1	Z2	0.5
Z2	Z3	3.4
Z1	Z3	2.2

Example 1 – Wiring Network

This section presents an example of how to create a minimum spanning tree from the data presented in the example given above. The data are contained in the *Wiring Network* database. Each row represents a possible arc (connection or branch) of the network by identifying the end points along with the length between them.

Wiring Network Dataset

Site1	Site2	Length
A1	A2	6.8
A1	A3	3.2
A2	B1	9.1
A2	A3	8.5
A2	B2	7.2
B1	B2	10.3
A3	B2	14.8
A3	C1	5.6
B2	C1	8.2
B2	C2	9.1
C1	C2	10.9
Z1	Z2	0.5
Z2	Z3	3.4
Z1	Z3	2.2

Setup

To run this example, complete the following steps:

1 Open the Wiring Network example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **Wiring Network** and click **OK**.

2 Specify the Minimum Spanning Tree procedure options

- Find and open the **Minimum Spanning Tree** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Specifications Tab

Node 1 Column...**Site1**
Node 2 Column...**Site2**
Length Column ....**Length**

3   Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Minimum Spanning Tree

Minimum Spanning Forest

Tree	Arc (Row)	Length (Length)	Site1 (Node 1)	Site2 (Node 2)
1	1	6.8	A1	A2
1	2	3.2	A1	A3
1	3	9.1	A2	B1
1	5	7.2	A2	B2
1	8	5.6	A3	C1
1	10	9.1	B2	C2
2	12	0.5	Z1	Z2
2	14	2.2	Z1	Z3
Total		43.7		

This report lists the solution by giving the arcs that form the minimum spanning tree(s). Note that in this example, there were two trees, so the solution is called a forest. You can see that there are no defined arcs between the first and second trees.

Node Connections

Node Connections

Node	Connected Nodes
A1	A2*, A3*
A2	A1*, B1*, A3, B2*
A3	A1*, A2, B2, C1*
B1	A2*, B2
B2	A2*, B1, A3, C1, C2*
C1	A3*, B2, C2
C2	B2*, C1
Z1	Z2*, Z3*
Z2	Z1*, Z3
Z3	Z2, Z1*

* The connected node is starred if the pair is in the minimum spanning tree.

This report presents a list of the nodes followed by the nodes they are connected with. If the pair of nodes is in the minimum spanning tree, the connected note is starred.

Possible Network Arcs

Possible Network Arcs

Tree	Arc (Row)	Length (Length)	Site1 (Node 1)	Site2 (Node 2)
1	1*	6.8	A1	A2
1	2*	3.2	A1	A3
1	3*	9.1	A2	B1
	4	8.5	A2	A3
1	5*	7.2	A2	B2
	6	10.3	B1	B2
	7	14.8	A3	B2
1	8*	5.6	A3	C1
	9	8.2	B2	C1
1	10*	9.1	B2	C2
	11	10.9	C1	C2
2	12*	0.5	Z1	Z2
	13	3.4	Z2	Z3
2	14*	2.2	Z1	Z3

* The starred arcs are in the minimum spanning tree.

This report lists the arcs that were input from the dataset. The started arcs are in the minimum spanning tree.