Chapter 254

R & R Study

Introduction

A repeatability and reproducibility (R & R) study (sometimes called a gauge study) is conducted to determine if a particular measurement procedure is adequate. If the measurement variation is small relative to the actual process variation, the measurement procedure is adequate. If it is not, the measurement procedure must be improved before it can satisfactorily monitor the process. For example, if your manufacturing specifications are in millimeters, but your measuring device provides readings only in centimeters, you are in trouble.

R & R studies separate process variation into that due to the measurement procedure and that due to the production process itself. The measurement variation is further divided into that due to the appraiser (reproducibility) and that due to the measuring device (repeatability).

It is important to emphasize that an R & R study is concerned with the precision of the measurement process. Data for R & R studies come from experiments especially designed for that purpose and that purpose only! Do not attempt to combine these studies with other experiments that you are conducting.

Several booklets are available that discuss R & R studies in detail. We recommend Barrentine (1991) and AIAG (1995). Although both of these concentrate on the "control chart" approach, they mention the analysis of variance approach—which we use here. The AIAG booklet states that the control chart approach is to be used only when software to analyze your data with the analysis of variance approach is not available.

The Analysis of Variance Approach to R & R

The analysis of variance model of this experimental design is

$$Y_{ijk} = \mu + P_i + O_j + (PO)_{ij} + E_{ijk}$$

where $i = 1, ..., I$; $j = 1, ..., J$; $k = 1, ..., K$; $I = 10$; $J = 3$; $K = 3$; and $P_i, O_j, (PO)_{ij}, E_{ijk}$ are jointly independent normal random variables with means of zero and variances $\sigma_P^2, \sigma_O^2, \sigma_{PO}^2$, and σ_E^2, respectively. These variances are often referred to as *variance components*. We let S represent the samples (parts), O represent the operators (appraisers), and E represent the random error.

In terms of this model, repeatability is σ_E^2, reproducibility is $\gamma_1 = \sigma_O^2 + \sigma_{PO}^2$, and the total variability associated with the measurement procedure is $\gamma_2 = \sigma_O^2 + \sigma_{PO}^2 + \sigma_E^2$, which may be called the R & R value. The process (sample-to-sample) variability is represented by σ_P^2. A ratio that compares process variability to measurement variability is

$$\delta = \frac{\sigma_P^2}{\sigma_O^2 + \sigma_{PO}^2 + \sigma_E^2}$$

Several indices have been devised to summarize the results of such an R & R study. Many are based on the above quantities. For example, the automotive group defines the signal-to-noise ratio as

$$SNR = \sqrt{\delta} = \sqrt{\frac{\sigma_P^2}{\sigma_O^2 + \sigma_{PO}^2 + \sigma_E^2}}$$

and the number of distinct product categories that can be reliably distinguished by the measurement procedure as

$$Distinct\ Categories = \sqrt{2\delta} = \sqrt{\frac{2\sigma_P^2}{\sigma_O^2 + \sigma_{PO}^2 + \sigma_E^2}}$$

Two popular measures that compare the measurement variance to the tolerance, where tolerance is the difference between the upper specification limit (USL) and lower specification limit (LSL), are the *measurement error ratio*

$$M = \frac{3\sqrt{\sigma_O^2 + \sigma_{PO}^2 + \sigma_E^2}}{USL - LSL} \times 100\%$$

and the *precision-to-tolerance ratio* (P/T)

$$PT = \frac{6\sqrt{\sigma_O^2 + \sigma_{PO}^2 + \sigma_E^2}}{USL - LSL} \times 100\%$$

All of these quantities are estimated in the analysis of variance approach with confidence intervals as well as point estimates.

The goal of the analysis is to estimate these quantities and determine if they fall within previously set guidelines.

Data Structure

Burdick and Larsen (1997) discuss an R & R study conducted to determine the capability of a procedure for monitoring the chemical content of a large tank. Ten samples are taken from the tank. A random sample of three operators is selected for the study. Each operator measures the chemical content of each of the ten samples three times using the same measurement device. The operator's measurements are made in random order. It is assumed that the operators are experienced so that no learning occurs during the study. The ninety values of acid concentration are recorded in the RRSTUDY dataset and displayed in the following table. Note that the results of a particular trial (a measurement by each of the three operators) are recorded a single row. Since each sample is measured three times by each operator, the results for each sample use three rows of the dataset.

An alternate way of entering these data is given at the end of this chapter.

RRStudy Dataset

Sample	Op1	Op2	Op3
1	67	66	69
1	68	68	67
1	68	68	68
2	67	67	67
2	66	67	66
2	66	68	66
3	68	70	68
3	68	70	68
3	67	68	68
4	67	70	67
4	67	68	68
4	67	70	68
5	68	70	69
5	68	70	68
5	68	70	69
6	69	71	70
6	68	70	70
6	69	70	70
7	67	68	68
7	67	68	68
7	67	68	69
8	75	75	75
8	74	75	75
8	74	75	75
9	67	69	68
9	67	68	68
9	67	69	68
10	66	68	66
10	66	66	66
10	66	66	66

Missing Values

Missing values are not allowed in this analysis. The confidence limits are based on formulas for experiments in which no data are missing. If you have missing values, you should resolve them by removing the measurements for the sample with the missing data from the analysis. The bottom line is this—make sure you do not allow missing values!

Example 1 – Running an R & R Study

This section presents an example of how to run an R & R study of the data that were displayed earlier in this chapter. These data are contained in the RRStudy dataset. In this example, ten chemical samples were selected for analysis. Each of three operators measured each of the ten samples three times. Each row contains one of the three trials for a particular sample. A trial consists of a measurement by each operator.

Setup

To run this example, complete the following steps:

1   Open the RRStudy example dataset

- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **RRStudy** and click **OK**.

2   Specify the R & R Study procedure options

- Find and open the **R & R Study** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 1** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Variables Tab

Sample (Part) Variable ....**Sample**
Measurement Variable(s) **Op1-Op3**
Lower Spec Limit **48**
Upper Spec Limit ....**88** (Note that 88 - 48 = 40 which is the tolerance.)
Target Spec **68**

3   Run the procedure

- Click the **Run** button to perform the calculations and generate the output.

Data Summary Section

Data Summary Section

Item	Actual Count	Expected Count
Total Values	90	90
Sample	10	
Operators	3	
Replicates	3	

This section presents a summary of the number of data values analyzed. In order for the analysis to be valid, the Actual Count must match the Expected Count in the Total Values row. When this occurs, the design (data matrix) is said to be balanced. All of the formulas used are for balanced data matrices only.

Total Values

The number of nonmissing data values in the dataset. If the design is balanced, the entry on this line equals the product of the entries on the next three lines.

Samples

The number of samples (parts) found in the dataset.

Appraisers

The number of appraiser (operator) variables selected.

Replicates

The number of times an operator measured the same part.

Expected Mean Square and Variance Component Section

Expected Mean Square and Variance Component Section

Source Term	DF	Expected Mean Square	Variance Component	Lower 90% Conf. Limit	Upper 90% Conf. Limit
Sample (P)	9	R+3(PO)+9(P)	5.615638	2.948817	15.33656
Operators (O)	2	R+3(PO)+30(O)	0.3563786	0.1016096	7.389713
Interaction (PO)	18	R+3(PO)	0.1251029	0.02385001	0.3455315
Replicates (R)	60	R	0.3444445	0.2613323	0.4785284

The expected mean square expressions and variance components are for each term in the analysis of variance model.

Source Term

The source of variation or term in the model.

DF

The degrees of freedom. The number of observations "used" by this term.

Expected Mean Square

This is the symbolic value of the mean square for the term in the ANOVA model assuming balanced data (equal group counts). "P" represents σ_P^2. "O" represents σ_O^2. "PO" represents σ_{PO}^2. "R" represents σ_E^2.

Variance Component

This is the expected value of corresponding variance in the ANOVA model assuming balanced data (equal group counts). Hence, the estimate of σ_P^2 is 5.615638 and the estimate of σ_{PO}^2 is 0.1251029. The formulas used for these estimates are

$$\sigma_O^2 = \frac{MS_O - MS_{PO}}{IK}$$

$$\sigma_P^2 = \frac{MS_P - MS_{PO}}{JK}$$

$$\sigma_{PO}^2 = \frac{MS_{PO} - MS_E}{K}$$

$$\sigma_E^2 = MS_E$$

where MS_Q represents the mean square of term Q in an analysis of variance table.

Lower (and Upper) Conf. Limit

These are the lower and upper confidence limits (interval estimate) of the variance components. The formulas used are found in Burdick and Larsen (1997). They are given as follows:

Confidence Interval for σ_O^2 is

$$Lower_O = \frac{MS_O - MS_{PO} - \sqrt{G_O^2 MS_O^2 + H_{PO}^2 MS_{PO}^2 + G_{O,PO} MS_O MS_{PO}}}{IK}$$

$$Upper_O = \frac{MS_O - MS_{PO} + \sqrt{H_O^2 MS_O^2 + G_{PO}^2 MS_{PO}^2 + H_{O,PO} MS_O MS_{PO}}}{IK}$$

Confidence Interval for σ_P^2 is

$$Lower_P = \frac{MS_P - MS_{PO} - \sqrt{G_P^2 MS_P^2 + H_{PO}^2 MS_{PO}^2 + G_{P,PO} MS_P MS_{PO}}}{JK}$$

$$Upper_P = \frac{MS_P - MS_{PO} + \sqrt{H_P^2 MS_P^2 + G_{PO}^2 MS_{PO}^2 + H_{S,PO} MS_P MS_{PO}}}{JK}$$

Confidence Interval for σ_{PO}^2 is

$$Lower_{PO} = \frac{MS_{PO} - MS_E - \sqrt{G_{PO}^2 MS_{PO}^2 + H_E^2 MS_E^2 + G_{PO,E} MS_{PO} MS_E}}{K}$$

$$Upper_{PO} = \frac{MS_{PO} - MS_E + \sqrt{H_{PO}^2 MS_{PO}^2 + G_E^2 MS_E^2 + H_{PO,E} MS_{PO} MS_E}}{K}$$

Confidence Interval for σ_E^2 is

$$Lower_E = (1 - G_E) MS_E$$

$$Upper_E = (1 + H_E) MS_E$$

where

$$G_q = 1 - \frac{1}{F_{\alpha, n_q, \infty}}$$

$$H_q = \frac{1}{F_{1-\alpha, n_q, \infty}} - 1$$

$$G_{qr} = \frac{\left(F_{\alpha, n_q, n_r} - 1\right)^2 - G_q^2 F_{\alpha, n_q, n_r}^2 - H_r^q}{F_{\alpha, n_q, n_r}}$$

$$H_{qr} = \frac{\left(1 - F_{1-\alpha, n_q, n_r}\right)^2 - H_q^2 F_{1-\alpha, n_q, n_r}^2 - G_r^q}{F_{1-\alpha, n_q, n_r}}$$

and F_{α, n_q, n_r} is the F distribution with an area equal to α to the right. The confidence level of these intervals is $100(1 - 2\alpha)\%$. The subscripts q and r refer to the terms *O, P, PO,* and *E.* The *n's* are given by

$$n_s = I - 1$$

$$n_O = J - 1$$

$$n_{SO} = (I - 1)(J - 1)$$

$$n_E = IJ(K - 1)$$

Analysis of Variance Section

Analysis of Variance Section

Source Term	DF	Sum of Squares	Mean Square	F-Ratio	Prob Level
Sample	9	461.3445	51.26049	71.22*	0.000000
Operators	2	22.82222	11.41111	15.85*	0.000107
Interaction	18	12.95556	0.7197531	2.09*	0.017450
Replicates	60	20.66667	0.3444445		
Total (Adjusted)	89	517.7889			
Total	90				

Source Term

The source of variation. The term in the model.

DF

The degrees of freedom. The number of observations "used" by the corresponding model term.

Sum of Squares

This is the sum of squares for this term. It is usually included in the ANOVA table for completeness, not for direct interpretation.

Mean Square

An estimate of the variation accounted for by this term. The sum of squares divided by the degrees of freedom.

F-Ratio

The ratio of the mean square for this term and the mean square of its corresponding error term. This is also called the F-test value.

Prob Level

The significance level of the above F-ratio. The probability of an F-ratio larger than that obtained by this analysis. For example, to test at an alpha level of 0.05, this probability would have to be less than 0.05 to make the F-ratio significant. Note that if the value is significant at the specified value of alpha, a star is placed to the right of the F-Ratio.

Variance Section

Variance Section

Term	Variance	% Total Variance	Standard Deviation	Lower 90% Conf. Limit of Std Dev	Upper 90% Conf. Limit of Std Dev	% Total Std Dev
Sample	5.615638	87.1782	2.3697	1.7172	3.9162	93.3693
Operators	0.356379	5.5325	0.5970	0.3188	2.7184	23.5212
Interaction	0.125103	1.9421	0.3537	0.1544	0.5878	13.9360
Reproducibility	0.481481	7.4746	0.6939	0.4349	2.7415	27.3397
Repeatability	0.344444	5.3472	0.5869	0.5112	0.6918	23.1241
R and R	0.825926	12.8218	0.9088	0.7443	2.8044	35.8076
Total Variation	6.441564	100.0000	2.5380	1.9394	4.2947	100.0000

This report presents estimates of the variance and standard deviation of various terms of interest in an R & R study.

Term

These are the names of the variance terms being estimated. The first few terms were discussed above in the Expected Mean Square and Variance Component Report. "Sample" refers to σ_P^2, the variability between samples (parts). "Operators" refers to σ_O^2, the variability between appraisers (operators). "Interaction" refers to σ_{PO}^2, the interaction variation. "Repeatability" refers to σ_E^2, the variability that occurs when one appraiser measures the same sample over and over.

"Reproducibility" refers to the variation among appraisers which is $\gamma_1 = \sigma_O^2 + \sigma_{PO}^2$. "R and R" refers to the sum of Reproducibility and Repeatability which is $\gamma_2 = \sigma_E^2 + \sigma_O^2 + \sigma_{PO}^2$. "Total Variation" is the sum of all four sources of variation $\sigma_T^2 = \sigma_E^2 + \sigma_P^2 + \sigma_O^2 + \sigma_{PO}^2$.

Variance

These are the estimated values of the variances of the terms listed above. The formulas for the first four terms were given in the Expected Mean Square and Variance Component Report. The formulas for the last three items are as follows.

$$\hat{\gamma}_1 = \frac{MS_O + (I-1)MS_{PO} - I(MS_E)}{IK}$$

$$\hat{\gamma}_2 = \frac{MS_O + (I-1)MS_{PO} + I(K-1)MS_E}{IK}$$

$$\hat{\sigma}_T^2 = \frac{MS_P}{(I-1)(J-1)^2} + \frac{MS_O}{(I-1)^2(J-1)} + \left[\frac{(I-1)(J-1) - (I-1) - (J-1)}{(I-1)^2(J-1)^2}\right]MS_{PO}$$

$$+ \left[\frac{(I-1)(J-1) - 1}{(I-1)(J-1)}\right]MS_E$$

% Total Variance

This shows the percentage that each variance is of the Total Variation variance.

Standard Deviation

This is the square root of the variance.

Lower (and Upper) 90% Conf. Limit

These are the lower and upper confidence limits (interval estimate) for the standard deviation shown in the previous column. The formulas used are found by taking the square root of the corresponding variance confidence limits found in Burdick and Larsen (1997). The values of the first four terms were given in the Expected Mean Square and Variance Component Report. The formulas for the last three items are as follows.

Confidence Interval for Reproducibility, $\sigma_O^2 + \sigma_{PO}^2$, is

$$Lower_{\gamma_1} = \hat{\gamma}_1 - \sqrt{V_{L\gamma}}$$

$$Upper_{\gamma_1} = \hat{\gamma}_1 + \sqrt{V_{U\gamma}}$$

where

$$V_{L\gamma} = \frac{\begin{array}{c} G_O^2 MS_O^2 + G_{PO}^2(I-1)^2 MS_{PO}^2 + H_E^2 I^2 MS_E^2 + G_{O,E}(I)MS_O MS_E + G_{PO,E} I(I-1)MS_{PO} MS_E \\ + G_{O,PO}^*(I-1)MS_O MS_{PO} \end{array}}{(IK)^2}$$

$$V_{U\gamma} = \frac{H_O^2 MS_O^2 + H_{PO}^2(I-1)^2 MS_{PO}^2 + G_E^2 I^2 MS_E^2 + H_{O,E}(I)MS_O MS_E + H_{PO,E} I(I-1)MS_{PO} MS_E}{(IK)^2}$$

and

$$G_{O,PO}^* = \left(1 - \frac{1}{F_{\alpha,I(J-1),\infty}}\right)^2 \frac{I^2}{I-1} - \frac{G_O^2}{I-1} - (I-1)G_{PO}^2$$

Confidence Interval for R and R, $\sigma_O^2 + \sigma_{PO}^2 + \sigma_E^2$, is

$$Lower_{\gamma_2} = \hat{\gamma}_2 - \sqrt{V_{LRR}}$$

$$Upper_{\gamma_2} = \hat{\gamma}_2 + \sqrt{V_{URR}}$$

where

$$V_{LRR} = \frac{G_O^2 MS_O^2 + G_{PO}^2(I-1)^2 MS_{PO}^2 + G_E^2 I^2(K-1)^2 MS_E^2}{(IK)^2}$$

$$V_{URR} = \frac{H_O^2 MS_O^2 + H_{PO}^2(I-1)^2 MS_{PO}^2 + G_E^2 I^2(K-1)^2 MS_E^2}{(IK)^2}$$

Confidence Interval for Total Variation, $\sigma_P^2 + \sigma_O^2 + \sigma_{PO}^2 + \sigma_E^2$, is

$$Lower_T = \hat{\sigma}_T^2 - \sqrt{V_{LT}}$$

$$Upper_T = \hat{\sigma}_T^2 + \sqrt{V_{UT}}$$

where

$$V_{LT} = G_P^2 C_P^2 MS_P^2 + G_O^2 C_O^2 MS_O^2 + G_{PO}^2 C_{PO}^2 MS_{PO}^2 + G_E^2 C_E^2 MS_E^2$$

$$V_{UT} = H_P^2 C_P^2 MS_P^2 + H_O^2 C_O^2 MS_O^2 + H_{PO}^2 C_{PO}^2 MS_{PO}^2 + H_E^2 C_E^2 MS_E^2$$

and

$$C_P = \frac{1}{(I-1)(J-1)^2}$$

$$C_O = \frac{1}{(I-1)^2(J-1)}$$

$$C_{PO} = \frac{(I-1)(J-1) - (I-1) - (J-1)}{(I-1)^2(J-1)^2}$$

$$C_E = \frac{(I-1)(J-1) - 1}{(I-1)(J-1)}$$

% Total Std Dev

This column gives the percentage that each standard deviation is of the total standard deviation. Because the total standard deviation is not equal to the sum of the individual standard deviations (it is the variances that are summed), these percentages may total to more than 100.

Percent of Process Variation R & R Section

Percent of Process Variation R & R Section

Term	Lower 90% Conf. Limit	5.15 Std Dev	Upper 90% Conf. Limit	% Total Variation	Percent Contribution
Sample	8.8436	12.2041	20.1684	93.3693	87.1782
Operator	1.6416	3.0744	13.9998	23.5212	5.5325
Interaction	0.7953	1.8215	3.0273	13.9360	1.9421
Reproducibility	2.2395	3.5735	14.1187	27.3397	7.4746
Repeatability	2.6327	3.0225	3.5626	23.1241	5.3472
R and R	3.8332	4.6803	14.4425	35.8076	12.8218
Total Variation	9.9877	13.0708	22.1175	100.0000	100.0000

Since the % R&R value is greater than 30%, the measurement system is not acceptable. Identify the measurement problems and correct them.

This report gives components of the process variation scaled by multiplying by the Sigma Multiplier value (which defaults to 5.15). This multiplication puts all values in same metric as the specification limits so they can be compared directly. For example, the variability that occurs when the same appraiser measures the same sample twice adds between 2.6327 and 3.5626 to the measurement standard deviation. Hence, by comparing these values, we can see the relative impact of each source of variation.

Term

These are the names of the terms being displayed. All of these terms have been defined previously.

Lower (and Upper) 90% Conf. Limit

These are the lower and upper confidence limits (interval estimate) for the standard deviation shown in between these two columns. The formulas used are found by taking the square root of the corresponding variance confidence limits found in Burdick and Larsen (1997). The values are multiplied by the Sigma Multiplier as discussed above.

5.15 Std Dev

This is the square root of the variance associated with each term multiplied by the Sigma Multiplier (5.15 is the default).

% Total Variation

This is 100 times the ratio of this term's standard deviation to the total variation's standard deviation. One of the key statistics to look at is whether the R and R value in this column is small enough. If the R and R value is less than 10%, the measurement procedure is deemed excellent. When it is less than 20%, it is deemed adequate. When it is less than 30%, it is marginal. When the R and R value is greater than 30%, it should not be used for process monitoring.

Percent Contribution

This is 100 times the ratio of this term's variance to the total variation's variance.

Percent of Tolerance R & R Section

Percent of Tolerance R & R Section

Term	Lower 90% Conf. Limit	5.15 Std Dev	Upper 90% Conf. Limit	Percent Tolerance
Sample	8.8436	12.2041	20.1684	30.5103
Operator	1.6416	3.0744	13.9998	7.6860
Interaction	0.7953	1.8215	3.0273	4.5539
Reproducibility	2.2395	3.5735	14.1187	8.9338
Repeatability	2.6327	3.0225	3.5626	7.5563
R and R	3.8332	4.6803	14.4425	11.7009
Total Variation	9.9877	13.0708	22.1175	32.6771

Spec Limits and Tolerance

Upper Spec Limit	88
Lower Spec Limit	48
Tolerance	40

Since the % R&R value is between 10% and 20%, the measurement system is acceptable.

This report is similar to the last report, except that the denominator of the percentages in the last column is the tolerance rather than the total variation.

Term

These are the names of the terms being displayed. All of these terms have been defined previously.

Lower (and Upper) 90% Conf. Limit

These are the lower and upper confidence limits (interval estimate) for the standard deviation shown in between these two columns. The formulas used are found by taking the square root of the corresponding variance confidence limits found in Burdick and Larsen (1997). The values are multiplied by the Sigma Multiplier as discussed above.

5.15 Std Dev

This is the square root of the variance associated with each term multiplied by the Sigma Multiplier (5.15 is the default).

Percent Tolerance

This is 100 times 5.15 times the ratio of this term's standard deviation to the tolerance. One of the key statistics to look at is whether the R and R value in this column is small enough. If the R and R value is less than 10%, the measurement procedure is deemed excellent. When it is less than 20%, it is deemed adequate. When it is less than 30%, it is marginal. When the R and R value is greater than 30%, it should not be used for process monitoring.

Upper (Lower) Spec Limits and Tolerance

The upper and lower specification limits are specified by the user. The tolerance is the upper specification limit minus the lower specification limit.

R & R Indices Section

R & R Indices Section

Index	Lower 90% Conf. Limit	Value	Upper 90% Conf. Limit
Distinct Categories	1.1924	3.6876	6.2979
Signal-to-Noise Ratio	0.8431	2.6075	4.4533
Measurement Error	5.5823	6.8160	21.0328
Precision-to-Tolerance	11.1647	13.6321	42.0655

Since the lower confidence limit of Distinct Categories is less than 3, the measurement process may be inadequate.
Since the upper confidence limit of Measurement Error is less than 25%, measurement error can be ignored in decision making.

This report gives values with confidence limits for four indices that have been found useful in analyzing R & R data. You will have to decide whether to use the point estimate (the Value) or the interval estimate (the Confidence Limits) when making decisions.

The first three statistics on this report are based on the ratio

$$\delta = \frac{\sigma_P^2}{\sigma_O^2 + \sigma_{PO}^2 + \sigma_E^2}$$

Confidence limits for this ratio are given below.

Single-to-Noise Ratio

This index is given by the formula

$$SNR = \sqrt{\hat{\delta}} = \sqrt{\frac{\hat{\sigma}_P^2}{\hat{\sigma}_O^2 + \hat{\sigma}_{PO}^2 + \hat{\sigma}_E^2}}$$

As you can see, it is the ratio of the sample-to-sample standard deviation and the measurement (R and R) variation. As a manufacturer, we are really interested in the sample-to-sample variability. The measurement standard deviation estimates the noise that is added to the sample-to-sample variability by the approximate nature of the measurement system.

The *Measurement Systems Analysis Reference Manual* (AGIG 1995) recommend that this value exceed 2.12.

Distinct Categories

This index is the number of distinct product categories that can be reliably distinguished by the measurement procedure. Its formula is

$$Distinct\ Categories = \sqrt{2\hat{\delta}} = \sqrt{\frac{2\hat{\sigma}_P^2}{\hat{\sigma}_O^2 + \hat{\sigma}_{PO}^2 + \hat{\sigma}_E^2}}$$

The *Measurement Systems Analysis Reference Manual* (AGIG 1995) recommend that this value exceed 3.

Measurement Error

This index compares the measurement standard deviation to the tolerance, where tolerance is the difference between the upper specification limit (USL) and lower specification limit (LSL). The value is calculated using the formula

$$M = \frac{3\sqrt{\hat{\sigma}_O^2 + \hat{\sigma}_{PO}^2 + \hat{\sigma}_E^2}}{USL - LSL} \times 100\%$$

A rule-of-thumb is that this value should be less than 25% in order for the measurement system to be deemed adequate.

Precision-to-Tolerance

A slightly different version of the Measurement Error index is the Precision-to-Tolerance ratio (P/T) which is defined as

$$PT = \frac{6\sqrt{\hat{\sigma}_O^2 + \hat{\sigma}_{PO}^2 + \hat{\sigma}_E^2}}{USL - LSL} \times 100\%$$

Confidence Limits for Ratio

The first three statistics on this report are function of the ratio

$$\delta = \frac{\sigma_P^2}{\sigma_O^2 + \sigma_{PO}^2 + \sigma_E^2}$$

The formulae for confidence limits of this statistic are given by Burdick and Larsen (1997). They are included here for easy reference. The approximate $100(1 - 2\alpha)\%$ confidence limits are

$$[L_\delta; U_\delta]$$

where

$$U_\delta = \frac{I}{J}\left[\frac{B_U + \sqrt{Q_U}}{2A_U}\right]$$

$$L_\delta = \frac{I}{J}\left[\frac{B_L - \sqrt{Q_L}}{2A_L}\right]$$

$$Q_U = Max[0, B_U^2 - 4A_U C_U]$$

$$Q_L = Max[0, B_L^2 - 4A_L C_L]$$

$$A_U = (1 - G_O^2)MS_O^2 + (I - 1)^2(1 - G_{PO}^2)MS_{PO}^2 + I^2(K - 1)^2(1 - G_E^2)MS_E^2 + 2(I - 1)MS_O MS_{PO}$$
$$+ 2I(K - 1)MS_O MS_E + 2I(I - 1)(K - 1)MS_{PO} MS_E$$

$$A_L = (1 - H_O^2)MS_O^2 + (I - 1)^2(1 - H_{PO}^2)MS_{PO}^2 + I^2(K - 1)^2(1 - H_E^2)MS_E^2 + 2(I - 1)MS_O MS_{PO}$$
$$+ 2I(K - 1)MS_O MS_E + 2I(I - 1)(K - 1)MS_{PO}MS_E$$

$$B_U = -2(I - 1)(1 - G_{PO}^2)MS_{PO}^2 + (2 + H_{P,O})MS_P MS_O + (I - 1)(2 + H_{P,PO})MS_P MS_{PO}$$
$$+ I(K - 1)(2 + H_{P,E})MS_P MS_E - 2MS_O MS_{PO} - 2I(K - 1)MS_{PO}MS_E$$

$$B_L = -2(I - 1)(1 - H_{PO}^2)MS_{PO}^2 + (2 + G_{P,O})MS_P MS_O + (I - 1)(2 + G_{P,PO})MS_P MS_{PO}$$
$$+ I(K - 1)(2 + G_{P,E})MS_P MS_E - 2MS_O^2 MS_{PO} - 2I(K - 1)MS_{PO}MS_E$$

$$C_U = (1 - H_P^2)MS_P^2 + (1 - G_{PO}^2)MS_{PO}^2 - (2 + H_{P,PO})MS_P MS_{PO}$$

$$C_L = (1 - G_P^2)MS_P^2 + (1 - H_{PO}^2)MS_{PO}^2 - (2 + G_{P,PO})MS_P MS_{PO}$$

Means and Bias Section

Means and Bias Section

Term	Count	Mean	Deviation From Target
Overall	90	68.589	0.589
Sample			
1	9	67.667	-0.333
2	9	66.667	-1.333
3	9	68.333	0.333
4	9	68.000	0.000
.	.	.	.
.	.	.	.
.	.	.	.
Operators			
Op1	30	67.967	-0.033
Op2	30	69.200	1.200
Op3	30	68.600	0.600
Sample,Operators			
1,Op1	3	67.667	-0.333
1,Op2	3	67.333	-0.667
1,Op3	3	68.000	0.000
2,Op1	3	66.333	-1.667
2,Op2	3	67.333	-0.667
2,Op3	3	66.333	-1.667
.	.	.	.
.	.	.	.
.	.	.	.

The main purpose of this report is to acquaint you with the data and allow you to quickly find outliers. We will discuss more about outliers below.
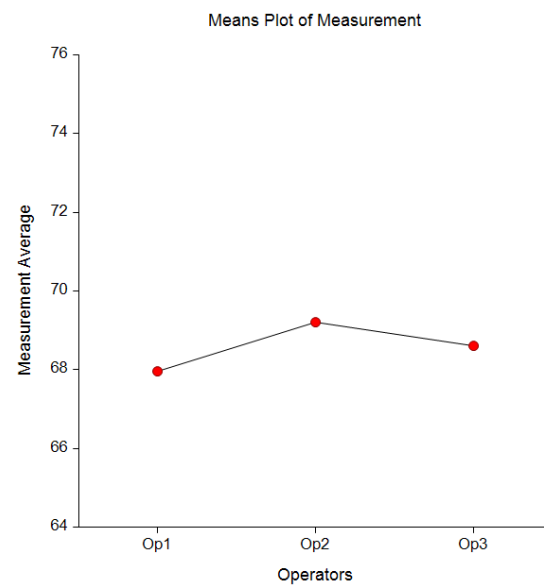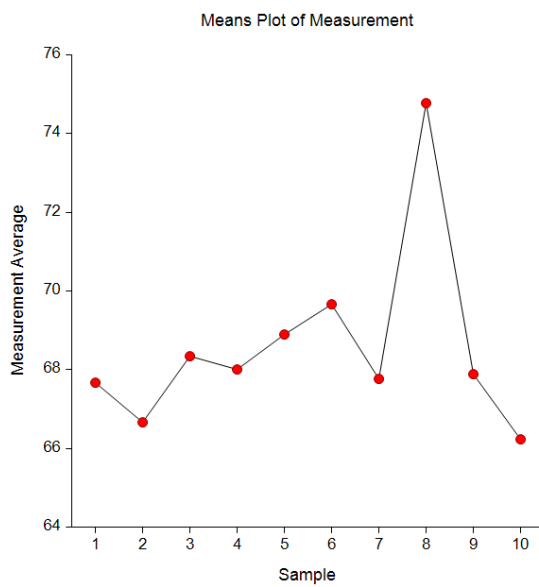
Term

The label for this line of the report.

Count
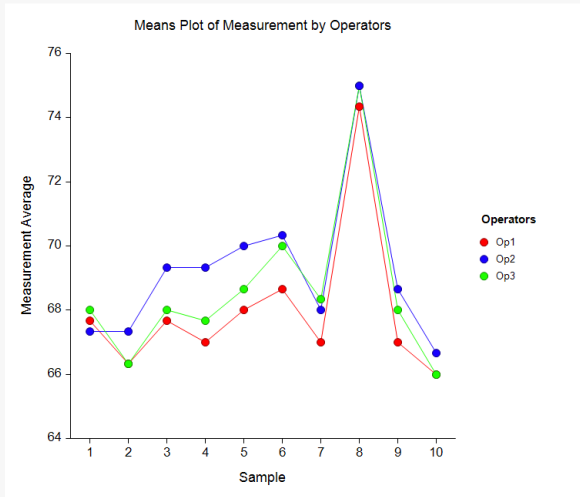
The number of observations in the mean.
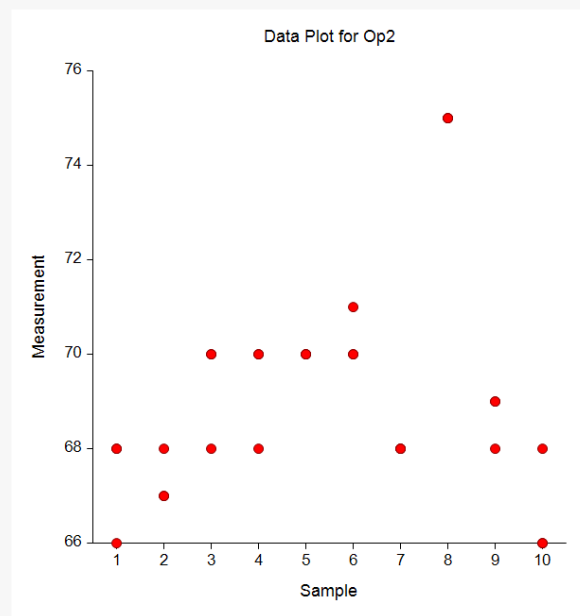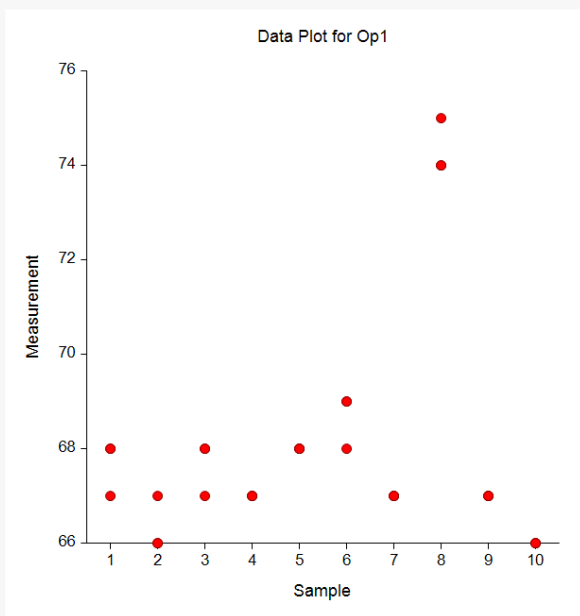
Mean

The value of the sample mean.

Bias

This is the difference between the mean and the Target Spec.

Plots Section

Means Plots

R & R Study
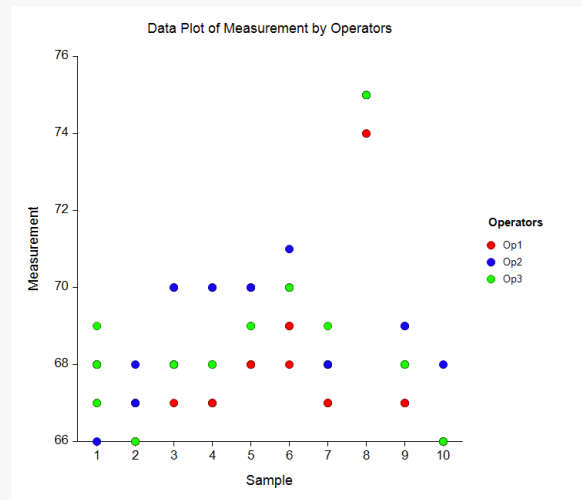

Means Plot of Measurement by Operators

Data Plots

Data Plot for Op1

Data Plot for Op2

R & R Study

Data Plot for Op3

Data Plot of Measurement by Operators
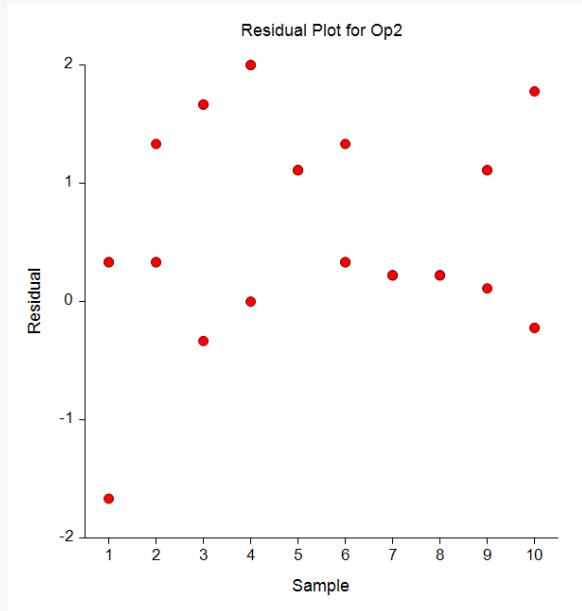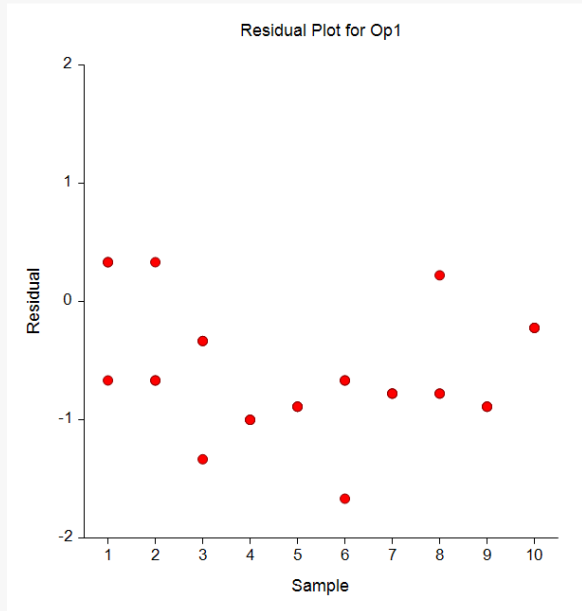
Operators
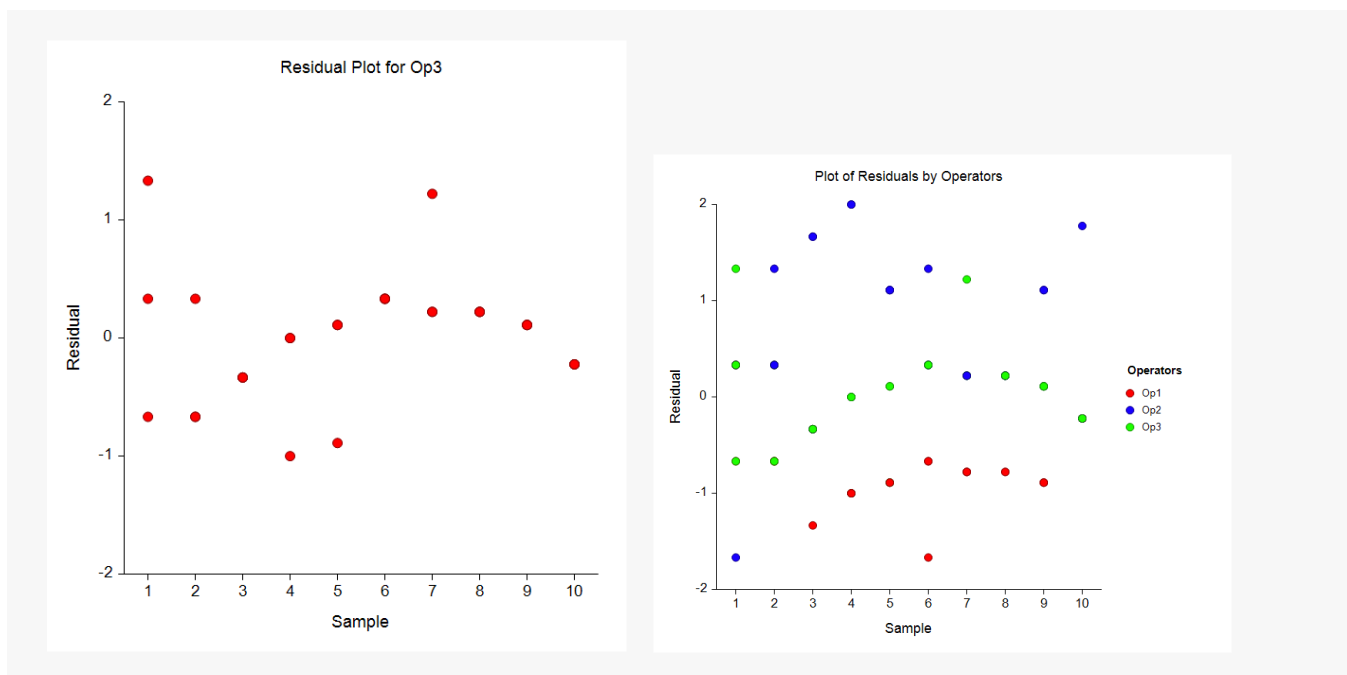- Op1
- Op2
- Op3

Residual Plots
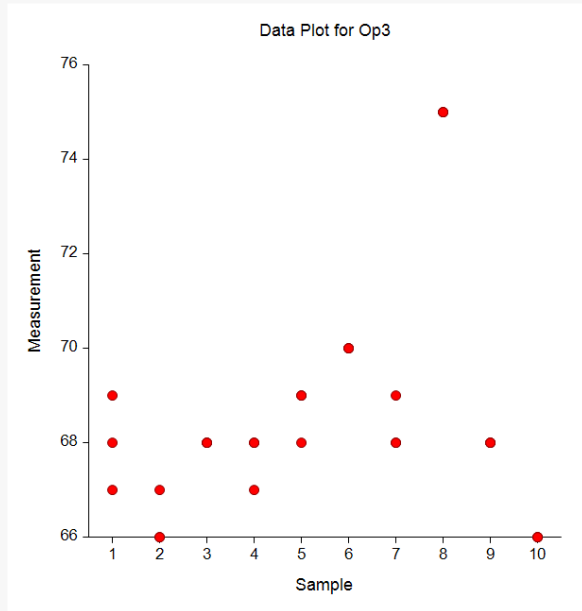
Residual Plot for Op1

Residual Plot for Op2

This section displays various plots of means, the original data, and residuals. You should look through these plots for unexpected patterns, trends, and outliers.

The plots of the means let you analyze the systematic variation in your data. For example, you can see whether one appraiser was very different from the rest. You can also determine whether certain samples were extremely different from the others.

The data plots let you see the original data. In these plots, you will be able to quickly find outliers (which often turn out to be data entry errors) and unusual patterns. This plot will give you a good feel for the variation in your data.

The residual plots show the deviation between each data value and the sample (part) mean for that value. This lets you view the measurement error.

Example 2 – Analysis of Variance Data

In this example, the RRStudy dataset has been reformatted to match the more typical data format necessary to run an analysis of variance on the data. The difference is that the operator factor is explicitly represented as a variable and only one measurement is given per row. This format requires ninety rows instead of thirty. The first six rows are displayed here. The complete dataset is contained in the RRStudy1 dataset.

RRStudy1 Dataset (Subset)

Sample	Operator	Measurement
1	Op1	67
1	Op1	68
1	Op1	68
2	Op1	67
2	Op1	66
2	Op1	66

Setup

To run this example, complete the following steps:

1   Open the RRStudy1 example dataset
- From the File menu of the NCSS Data window, select **Open Example Data**.
- Select **RRStudy1** and click **OK**.

2   Specify the R & R Study procedure options
- Find and open the **R & R Study** procedure using the menus or the Procedure Navigator.
- The settings for this example are listed below and are stored in the **Example 2** settings file. To load these settings to the procedure window, click **Open Example Settings File** in the Help Center or File menu.

Variables Tab

Sample (Part) Variable ....**Sample**

Appraiser (Operator) Variable.........................**Operator**

Measurement Variable(s) ....**Measurement**

Lower Spec Limit **48**

Upper Spec Limit ....**88** (Note that 88 - 48 = 40 which is the tolerance.)

Target Spec **68**

3   Run the procedure
- Click the **Run** button to perform the calculations and generate the output.

Since the same data are being analyzed, the reports are the same as in Example 1.